cation by Dalzell in 1861 of his "Flora of Bombay." It is impossible in a brief review like the present to mention the names of all the workers who, in various parts of the gradually extending Indian Empire, added to our knowledge of its botanical wealth. It must suffice to mention the names of a few of the chief, such as Hardwicke, Madden, Munro, Edgeworth, Lance and Vicary, who collected and observed in Northern India, and who all, except the two last mentioned, also published botanical papers and pamphlets of more or less importance; Jenkins, Masters, Mack, Simons and Oldham, who all collected extensively in Assam; Hofmeister, who accompanied Prince Waldemar of Prussia, and whose collections form the basis of the fine work by Klotsch and Garcke (*Reis. Pr. Wald.*); Norris, Prince, Lobb and Cuming, whose labours were in Penang and Malacca; and last, but not least, Strachey and Winterbottom, whose large and valuable collections, amounting to about 2000 species, were made during 1848 to 1850 in the higher ranges of the Kamaon and Gharwal Himalaya, and in the adjacent parts of Tibet. In referring to the latter classic Herbarium, Sir Joseph Hooker remarks that it is "the most valuable for its size that has ever been distributed from India." General Strachey is the only one who survives of the splendid band of collectors whom I have mentioned. I cannot conclude this brief account of the botanical labours of our first period without mentioning one more book, and that is the "Hortus Calcuttensis" of Voigt. Under the form of a list, this excellent work, published in 1845, contains a great deal of information about the plants growing near Calcutta, either wild or in fields and gardens. It is strong in vernacular names and vegetable economics.

(*To be continued.*)

MATHEMATICS AT THE BRITISH ASSOCIATION.

THE visit of the French Association to Dover necessitated some departures from the usual programme of the British Association week, and the mathematical meeting was held this year on Monday, September 18. Prof. Forsyth, of Cambridge, presided over a well-filled room.

The session opened with the formal communication of two reports of committees: the first, drawn up by Prof. Karl Pearson, and practically forming a continuation of a previous report, contains a set of tables of certain functions connected with the integral

$$G(r, \nu) = \int_0^\pi \sin^r\theta e^{\nu\theta} d\theta,$$

for integral values of r from 1 to 50, and for values of ν at certain intervals from 0 to 1. These functions are of importance in certain statistical problems.

The second report consists substantially of the new "Canon Arithmeticus" which Lieut.-Colonel Cunningham has prepared; the Association has made a grant for publishing the tables as a separate volume (they cannot well be fitted into the comparatively small page of the B.A. Report), and it is to be hoped that before long they will become generally available for workers in the Theory of Numbers.

The first of the papers was read by Dr. Francis Galton, on "The Median Estimate." Dr. Galton proposes to substitute a scientific method for the very unsatisfactory ways in which the collective opinion of committees and assemblies of various kinds is ascertained, in respect to the most suitable amount of money to be granted for any particular purpose. How is that medium amount to be ascertained which is the fairest compromise between many different opinions? An average value—*i.e.* the arithmetic mean of the different estimates—may greatly mislead, because a single voter is able to produce an effect far beyond his due share by writing down an unreasonably large or unreasonably small sum. Again, few persons know what they want with sufficient clearness to enable them to express it in numerical terms, from which alone an average may be derived; much deeper thought-searching is needed to enable a man to make such a precise affirmation as that "in my opinion the bonus to be given should be 80*l.*," than to enable him to say "I do not think he deserves so much as 100*l.*, certainly not more than 100*l.*"

Dr. Galton's plan for discovering the medium of the various sums desired by the several voters is to specify any two reasonable amounts A and B, and to find what percentage a of voters think that the sum ought to be less than A, and what percentage b vote for less than B. It may now be assumed that the estimates will be distributed on either side of their (unknown) *median* m, with an (unknown) quartile q, in approximate accordance with the normal law of frequency of error; and thus (using the table of centiles given in the author's "Natural Inheritance") the required median value can be found.

This was followed by a paper "On a system of invariants for parallel configurations in space," by Prof. Forsyth. The process followed by the author is one in which English mathematicians have always excelled—namely, the deduction of difficult analytical results from simple geometrical considerations. Prof. Forsyth's final formulæ may be regarded as invariantive relations between certain definite integrals; the way in which he finds them is as follows:—

Consider any plane curve; if we suppose a circle of constant size to roll on the curve, its envelope will be another curve, which is said to be *parallel* to the original one. If now L be the length and A the area of a curve, it is found that the quantity $A - \frac{1}{4\pi}L^2$ has the same value for the parallel as for the original curve; in other words,

$$A - \frac{1}{4\pi}L^2$$

is *invariantive* for parallel curves. Similarly in space of three dimensions, the envelope of a sphere of fixed size which rolls on a given surface is another *parallel* surface; and if V be the volume contained by a surface, S its superficial area, and L twice the surface-aggregate of the mean of the curvatures at any point, then it is found that the quantities

$$S - \frac{1}{16\pi}L^2 \text{ and } V - \frac{1}{8\pi}LS + \frac{1}{192\pi^2}L^3$$

are invariantive for all parallel surfaces.

Similar results hold for space of n dimensions. At the end of the paper the expressions obtained are shown to be connected with the ordinary invariant-theory of binary forms.

The next paper, read by Prof. Everett, was concerned with "The Notation of the Calculus of Differences." In conjunction with the ordinary symbol Δ, defined by

$$\Delta y_n = y_{n+1} - y_n,$$

Prof. Everett employs another symbol δ, defined by

$$\delta y_n = y_n - y_{n-1},$$

so that

$$\delta = \Delta/_{1+\Delta}.$$

The use of δ simplifies some of the well-known formulæ of the calculus of finite differences.

Prof. A. C. Dixon, of Galway, followed, with a paper "On the Partial Differential Equation of the Second Order." Let z be the dependent, and x and y the independent, variables; and with the usual notation, let

$$p = \frac{\partial z}{\partial x}, \quad q = \frac{\partial z}{\partial y}, \quad r = \frac{\partial z^2}{\partial x^2}, \quad s = \frac{\partial z^2}{\partial x \partial y}, \quad t = \frac{\partial z^2}{\partial y^2},$$

and consider the differential equation

$$f(x, y, z, p, q, r, s, t) = 0.$$

This may be supposed solved by using two more relations

$$u = a, \ v = b,$$

among the quantities x, y, z, p, q, r, s, t, to give values of r, s, t, which, when substituted in

$$dz = pdx + qdy, \quad dp = rdx + sdy, \quad dq = sdx + tdy,$$

render these three equations integrable. This will not be possible, of course, unless the expressions u, v, fulfil certain conditions. Prof. Dixon considers the case in which u can be so determined that v is only subjected to one condition, and finds that then du is a linear combination of the differential expressions used in Hamburger's method of solution. If such a function u can be found, the system $f=0$, $u=a$, will have a series of solutions depending on an arbitrary function of one variable, and involving two further arbitrary constants.

The next paper, "On the Fundamental Differential Equations of Geometry," was read by Dr. Irving Stringham, of the University of California. Dr. Stringham derives the analytical formulæ for non-Euclidian Geometry by following a procedure indicated by Feye St. Marie, and later discussed in Killing's "Nicht-Euclidischen Raumformen." Within an infinitesimal domain in non-Euclidian space, the propositions of Euclidian

Geometry may be regarded as true; from this fact can be deduced a group of equations typified by

$$\frac{da}{d\alpha}=\frac{f(b)}{\sin\gamma},\ \frac{db}{da}=\cos\gamma,\ f'(b)=-\frac{d\gamma}{da},$$

where a, b, c, are the sides of a triangle, and α, β, γ, the corresponding angles. From these, by appropriate eliminations and transformations, the differential equation

$$\{f(a)\}^2=-\kappa^2[1-\{f'(a)\}^2]$$

can be found for the function f. Solving this, we have

$$f(a)=\kappa\sinh\frac{a}{\kappa},$$

and thence can derive the fundamental equations of non-Euclidian measurement.

$$\sinh\frac{a}{\kappa}\Big/\sin\alpha=\sinh\frac{b}{\kappa}\Big/\sin\beta=\sinh\frac{c}{\kappa}\Big/\sin\gamma.$$

This was followed by the communication of a Report on the Problem of Three Bodies, which Mr. E. T. Whittaker was commissioned to prepare at the Toronto meeting. In a general sketch of the results, Mr. Whittaker explained the transformation which has taken place in dynamical astronomy as a result of the researches of Newcomb, Hill, Lindstedt and Poincaré. Formerly the subject might be said to consist of two departments—the planetary and lunar theories; now the distinction between these was becoming less prominent, as the Problem of Three Bodies was treated in greater generality. Among the advances referred to were Dr. Hill's introduction of periodic orbits as a substitute for Keplerian ellipses in the first approximation to the solution, Newcomb's proof that the problem can be solved by series in which the time occurs only in the arguments of trigonometric functions, Poincaré's theorem that these series are only asymptotic expansions, and Bruns' result that the system possesses no algebraic integrals other than those already known.

A second paper by Prof. Forsyth, "On Singular Solutions of Ordinary Differential Equations," described some properties of the p-discriminant and c-discriminant of an ordinary differential equation of the first order. The two last papers on the list were "An Application and Interpretation of Infinitesimal Transformations," by Dr. E. O. Lovett, of Princeton University, N.J.; and "On Fermat's Numbers," by Lieut.-Colonel Cunningham. In the absence of their authors the papers were communicated by title, and the session was closed.

Looking at the papers as a whole, they were of just that character which makes the B.A. meeting useful to mathematicians; that is, they related not so much to abstruse continuations of well-known theories as to new and little-known subjects, suggestions of improved notations, reports on the recent progress of different branches of mathematics, and generally all those topics for which discussion at a real meeting is more important than the publication of a paper.

PHYSICS AT THE BRITISH ASSOCIATION.

THE attendance of physicists at Dover was rather smaller than usual, on account of the occurrence of the Volta Centenary celebrations at Como and the simultaneous meetings of the French Association for the Advancement of Science at Boulogne. Several of those who in past years have been leaders in the discussions of Section A were this year conspicuous by their absence. Nevertheless, the papers read maintained a high standard of excellence, and the reports presented indicate that good work is being done by the committees appointed for scientific research.

The address delivered by Prof. Poynting, as President of the Section, was the subject of many conversations, not only among physicists but with biologists also; the existence of the sharp line which he indicated between the psychical and physical methods and the phenomena to which each is applicable, was acknowledged on all sides. The physicists were divided on the question of the danger of too much hypothesis, and especially on the possibility of the propagation of electromagnetic waves in air being due to the air as much as to the ether. All, however, were agreed in the expression of thanks to the President, proposed by Sir George Stokes and seconded by Sir Norman Lockyer.

In a paper on the spectroscopic examination of contrast phenomena, Mr. G. J. Burch described experiments which lend great support to the Young-Helmholtz theory of colour-vision. If the eye is fatigued by exposure to a very intense red light, such as sunlight filtered through red screens and focussed on the eye, and a spectrum be then looked at, the red is invisible; but the rest of the spectrum, green to violet, appears in its ordinary colours. Red-blindness is therefore not accompanied by green-blindness, as Hering's theory requires. Further experiments on the blue and violet portions of the spectrum have led Mr. Burch to the conclusion that we have separate primary sensations for blue and violet, in addition to those for red and green, making four altogether instead of the three postulated by the Young-Helmholtz theory. The experiments are the more convincing because carried out with spectral colours, thus avoiding all errors due to the impurity of pigment colours. In the discussion on the paper several members took part; Sir George Stokes said experiments led him to believe that lobelia blue is a primary sensation, and Principal Glazebrook suggested that the theory should be tested by colour-matches on a spectro-photometer.

Prof. Callendar gave the preliminary results of a research on the variation of the specific heat of water with temperature, which he commenced in Montreal with Mr. H. T. Barnes, and which is now being continued by the latter. The method of experiment consists in allowing water to flow steadily through a narrow tube along which a platinum wire runs axially; on passing a constant electric current through the wire the water finally acquires a steady temperature-difference between the inlet and outlet of the tube, which is measured by platinum thermometers and automatically recorded. Radiation corrections are reduced to a minimum by surrounding the tube with a vacuum-jacket, and the electrical energy supplied is measured by observing the current and the potential-difference between the ends of the wire in the tube. The results show that the specific heat of water has a minimum value of 0·995 in the neighbourhood of 40° C., it rises to 1·000 as the temperature falls to 10° C., and continues to rise rapidly as the temperature decreases. On increasing the temperature above 40° C. the specific heat rises to 0·997 at 60° C. Further experiments will be made in the neighbourhood of the freezing point and on either side of it.

The committee on electrolysis and electro-chemistry has undertaken the comparison of the variation of electrical conductivity with concentration, and the variation of freezing point with concentration for identical very dilute aqueous solutions of electrolytes. The electrical measurements have been successfully carried out by Mr. W. C. Whetham, but the freezing point determinations, undertaken by Mr. E. H. Griffiths, have been delayed by the discovery of errors arising from the presence of dissolved gases in the solutions. Incidentally Mr. Griffiths remarked that he was able to measure temperatures to within three or four parts in a million.

Dr. R. A. Lehfeldt, at a subsequent meeting, called attention to a flaw in Nernst's theory of electrolytic solution pressure. According to this theory, when a metal is immersed in an electrolyte ions are torn either from the metal or from the solution according as the solution-pressure is greater or less than the osmotic pressure of the ions in solution. It is usually supposed that the mass of the ions deposited or dissolved is so extremely small that it cannot be detected; the author showed, however, by considering the electrostatic tension due to the ionic charges, that the amount dissolved should be easily weighable, at any rate in the case of zinc.

The stability of an ether containing long, thin, empty vortex filaments was discussed in a communication by Prof. FitzGerald on the energy per cubic centimetre in a turbulent liquid transmitting laminar waves. Lord Kelvin considered this subject in 1887, and concluded that rapid diffusion would make the structure unstable. The author held the opinion (though possibly Lord Kelvin would differ from him) that the turbulency of a sufficiently fine-grained irregularly turbulent liquid would ultimately diffuse so slowly that Lord Kelvin's investigation could be applied to it.

Until the meeting of the Association in 1893, it was generally supposed that the absence of an atmosphere from the moon, and of hydrogen from our own atmosphere, is due to the high average velocity of the gaseous molecules, which is sufficient to carry them beyond the range of the moon's or earth's attraction. On that occasion Prof. Bryan demonstrated the incorrectness of this view for the case of the moon, and he has since extended his calculations to the cases of hydrogen and helium in the